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Analytical design of PID controller cascaded with a lead-lag filter for time-delay processes

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Abstract−An analytical method for the design of a proportional-integral-derivative (PID) controller cascaded with a second-order lead-lag filter is proposed for various types of time-delay process. The proposed design method is based on the IMC-PID method to obtain a desired, closed-loop response. The process dead time is approximated by using the appropriate Pade expansion to convert the ideal feedback controller to the proposed PID•filter structure with little loss of accuracy. The resulting PID•filter controller efficiently compensates for the dominant process poles and zeros and significantly improves the closed-loop performance. The simulation results demonstrate the superior performance of the proposed PID•filter controller over the conventional PID controllers. A guideline for the closed-loop time constant, λ, is also suggested for the FOPDT and SOPDT models.

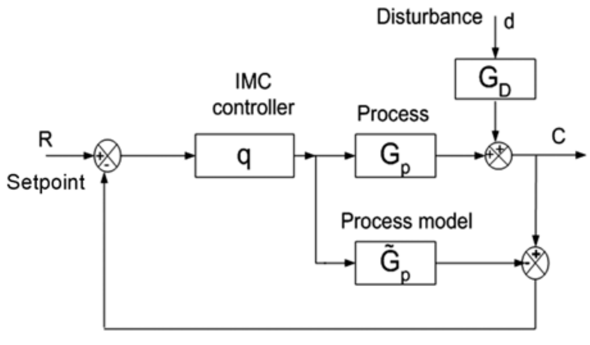
Key words: PID Controller, Lead-lag Filter, IMC-PID Design, Robust Analysis, Lag Time Dominant Process

INTRODUCTION

Due to their efficient and robust performance with a simple algorithm, proportional-integral-derivative (PID) controllers have gained wide acceptance in most industrial applications. Furthermore, recent developments of modern control systems have enabled the PID controller to be combined with various simple control algorithms in a quick and easy manner to enhance the control performance. To cascade a PID controller with a lead-lag filter is a typical example of taking advantage of the facility by using modern control systems. However, in spite of its significant potential to improve control performance, the PID•filter structure has not gained wide use in the process industries. The only popular trial so far is to add a first- or second-order lag filter in series with the PID controller for filtering high frequency noises. This limited application is mainly due to the lack of systematic design methods for the PID•filter control system.

Many previous studies have investigated the conventional PID controller design since Ziegler and Nichols [1] published their classical method. The internal model control (IMC)-based approach has gained widespread acceptance for the design of the PID controller in process industries because of its simplicity, robustness, and successful practical applications [2-10]. It also has a practical advantage in that a clear tradeoff between closed-loop performance and robustness is achieved with a single tuning parameter. In the IMC-PID methods, the PID controller parameters are obtained by first computing the ideal feedback controller that gives a desired closed-loop response. It is well known that most processes in the chemical industries can be represented by a first- or second-order plus dead time model, either with or without single zero. In the time-delay processes, the ideal feedback controllers are more complicated than the standard PID controllers. The controller form can then be reduced to that of a PID controller by clever approximations of the process dead time. However, all these methods concern the design of the conventional PID controller, while the control system with the PID controller often only shows a sluggish response and overshoots to the time-delay processes where lag time dominates.

In this study, an analytical method for the design of a PID controller cascaded with a second-order lead-lag filter is developed for various types of time-delay process. The proposed PID•filter controller is designed based on the IMC-PID principle and gives a better response than the conventional PID controllers reported in earlier studies for obtaining the desired, closed-loop response. Several examples are provided for comparing the results with the conventional PID controllers. In the proposed PID•filter structure, the result-



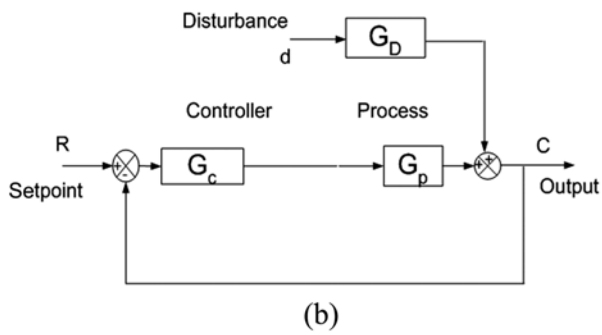


Fig. 1. Block diagram of IMC and classical feedback control.
(a) IMC structure, (b) Classical feedback control structure

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ing control system becomes equivalent to controlling a fast dynamic process by integral control, which significantly improves the performance. Some discussions with the λ guideline for particular robustness levels are also provided.

IMC-PID APPROACH FOR PID•FILTER CONTROLLER DESIGN

Figs. 1(a) and (b) show the block diagrams of IMC control and equivalent classical feedback control structures, respectively, where G_p is the process, $\tilde{G}_p$ the process model, q the IMC controller, and G_c the equivalent feedback controller. In the IMC control structure, the controlled variable is related as:

$$C=\frac{G_p q}{1+q(G_p-\tilde{G}_p)}R+\left[\frac{1-\tilde{G}_p q}{1+q(G_p-\tilde{G}_p)}\right]G_D d \tag{1}$$

For the nominal case (*i.e.*, $G_p=\tilde{G}_p$), the set-point and disturbance responses are simplified as:

$$\frac{C}{R}=\tilde{G}_p q \tag{2}$$

$$\frac{C}{d}=[1-\tilde{G}_p q]G_D \tag{3}$$

According to the IMC parameterization [3], the process model $\tilde{G}_p$ is factored into two parts:

$$\tilde{G}_p=P_M P_A \tag{4}$$

where P_M is the portion of the model inverted by the controller, P_A is the portion of the model not inverted by the controller and $P_A(0)=1$. The noninvertible part usually includes dead time and/or right half plane zeros and is chosen to be all-pass.

The IMC controller is designed by:

$$q=P_M^{-1}f \tag{5}$$

where the IMC filter f is usually set as:

$$f=\frac{1}{(\lambda s+1)^r} \tag{6}$$

The ideal feedback controller equivalent to the IMC controller can be expressed in terms of the internal model, $\tilde{G}_p$, and the IMC controller, q:

$$G_c=\frac{q}{1-\tilde{G}_p q}=\frac{P_M^{-1}}{(\lambda s+1)^r-P_A} \tag{7}$$

Suppose that the time-delay process can be represented by first- or second-order dynamics. The ideal feedback controller in Eq. (7) can be converted into the PID controller cascaded with a second-order lead-lag filter by using the following appropriate Pade approximations of the dead time:

$$G_c=K_c\left(1+\frac{1}{\tau_I s}+\tau_D s\right)\frac{1+cs+ds^2}{1+as+bs^2} \tag{8}$$

where K_c, τ_I and τ_D are the proportional gain, integral time constant, and derivative time constant of the PID controller, respectively, and a, b, c and d are the filter parameters. The second-order lead-lag filter is easily implemented by using modern control hardware. The PID•filter controller in Eq. (8) is an extension to the modified PID controller structure developed by Rivera et al. [2], Horn et al. [5], Lee et al. [6] Shamsuzzoha and Lee [8-10] and Dwyer [11].

1. First-order Plus Dead Time Process

The most commonly used approximate model for chemical processes is the first-order plus dead time (FOPDT) model as given below:

$$G_p=\frac{Ke^{-\theta s}}{\tau s+1} \tag{9}$$

The process model G_p is factored into two parts: $P_M=K/(\tau s+1)$ and $P_A=e^{-\theta s}$. The IMC controller is designed as $q=P_M^{-1}f=(\tau s+1)/K(\lambda s+1)$ for the desired closed-loop response, $e^{-\theta s}/(\lambda s+1)$. From Eq. (7), the ideal feedback controller is given by $G_c=(\tau s+1)/(K[\lambda s+1-e^{-\theta s}])$. Approximating the dead time $e^{-\theta s}$ with a 2/2 Pade expansion as $e^{-\theta s}=(1-\theta s/2+\theta^2 s^2/12)/(1+\theta s/2+\theta^2 s^2/12)$, we obtain the following tuning rule of the PID•filter controller after some simplification:

$$K_C=\frac{\theta}{2K(\lambda+\theta)},\ \tau_I=\frac{\theta}{2},\ \tau_D=\frac{\theta}{6},\ a=\frac{\theta\lambda}{2(\lambda+\theta)},$$
$$b=\frac{\theta^2\lambda}{12(\lambda+\theta)},\ c=\tau,\ d=0 \tag{10}$$

Note that the 2/2 Pade approximation is precise enough to convert the ideal feedback controller into a PID•filter controller with little loss of accuracy.

2. Second-order Plus Dead Time Process

Consider a general second-order plus dead time (SOPDT) process system,

$$G_p=\frac{Ke^{-\theta s}}{(\tau^2 s^2+2\tau\zeta s+1)} \tag{11}$$

where ζ is the damping factor. The process model is decomposed into $P_M=K/(\tau^2s^2+2\tau\zeta s+1)$ and $P_A=e^{-\theta s}$. The resulting IMC controller is then given as $q=P_M^{-1}f=(\tau^2s^2+2\tau\zeta s+1)/K(\lambda s+1)$ for the desired closed-loop response, $e^{-\theta s}/(\lambda s+1)$. From Eq. (7), the ideal feedback controller is given by $G_c=(\tau^2s^2+2\tau\zeta s+1)/(K[\lambda s+1-e^{-\theta s}])$. Substituting a 2/2 Pade approximation into the dead time term, we obtain the tuning rule of the PID•filter controller given in Table 1.

3. SOPDT Process with Overshoot Response

Consider a general SOPDT process with a lead term,

$$G_p=\frac{K(\tau_a s+1)e^{-\theta s}}{(\tau^2 s^2+2\tau\zeta s+1)} \tag{12}$$

The negative zero in the model causes an excessive overshoot in the open-loop response. Since the process model is factored into $P_M=K(\tau_a s+1)/(\tau^2s^2+2\tau\zeta s+1)$ and $P_A=e^{-\theta s}$, the IMC controller is given as $q=(\tau^2s^2+2\zeta\tau s+1)/K(\lambda s+1)(\tau_a s+1)$ for the desired closed-loop response $e^{-\theta s}/(\lambda s+1)$, while the equivalent ideal feedback controller is represented as $G_c=((1+\theta s/2+\theta^2s^2/12)(\tau^2s^2+2\tau\zeta s+1))/(K[(1+\theta s/2+\theta^2s^2/12)(\tau_a s+1)(\lambda s+1)-(\tau_a s+1)(1-\theta s/2+\theta^2s^2/12)])$ by replacing the dead time with a 2/2 Pade approximation. Neglecting all terms in the denominator higher than third-order by replacing $(1+\theta s/2+\theta^2s^2/12)(\tau_a s+1)(\lambda s+1)$ with $(1+\theta s/2)(\tau_a s+1)(\lambda s+1)$, we obtain the

Table 1. Design rules of the proposed PID•filter controller for various process models

Case	Process model	K_C	τ_I	τ_D	a	b	c	d
A	$\frac{Ke^{-\theta s}}{\tau s+1}$	$\frac{\theta}{2K(\lambda+\theta)}$	$\frac{\theta}{2}$	$\frac{\theta}{6}$	$\frac{\theta\lambda}{2(\lambda+\theta)}$	$\frac{\theta^2\lambda}{12(\lambda+\theta)}$	τ	-
B	$\frac{Ke^{-\theta s}}{(\tau^2 s^2+2\zeta s+1)}$	$\frac{\theta}{2K(\lambda+\theta)}$	$\frac{\theta}{2}$	$\frac{\theta}{6}$	$\frac{\theta\lambda}{2(\lambda+\theta)}$	$\frac{\theta^2\lambda}{12(\lambda+\theta)}$	τ^2	$2\zeta\tau$
C	$\frac{K(\tau_a s+1)e^{-\theta s}}{(\tau^2 s^2+2\zeta s+1)}$	$\frac{\theta}{2K(\lambda+\theta)}$	$\frac{\theta}{2}$	$\frac{\theta}{6}$	$\frac{[(\lambda+\tau_a)\theta/2+\tau_a\lambda-\theta^2/12+\tau_a\theta/2]}{(\theta+\lambda)}$	$\frac{\tau_a\theta\lambda/2-\tau_a\theta^2/12}{(\theta+\lambda)}$	τ^2	$2\zeta\tau$
D	$\frac{(-\tau_a s+1)Ke^{-\theta s}}{(\tau^2 s^2+2\zeta s+1)}$	$\frac{\theta}{2K(\lambda+\theta+2\tau_a)}$	$\frac{\theta}{2}$	$\frac{\theta}{6}$	$\frac{[(\lambda+\tau_a)\theta/2+\tau_a\lambda-\theta^2/12-\tau_a\theta/2]}{(\theta+\lambda+2\tau_a)}$	$\frac{\tau_a\theta\lambda/2+\tau_a\theta^2/12}{(\theta+\lambda+2\tau_a)}$	τ^2	$2\zeta\tau$
E	$\frac{K(-\tau_a s+1)e^{-\theta s}}{\tau s+1}$	$\frac{\theta}{2K(\lambda+\theta+2\tau_a)}$	$\frac{\theta}{2}$	$\frac{\theta}{6}$	$\frac{[(\lambda+\tau_a)\theta/2+\tau_a\lambda-\theta^2/12-\tau_a\theta/2]}{(\theta+\lambda+2\tau_a)}$	$\frac{\tau_a\theta\lambda/2+\tau_a\theta^2/12}{(\theta+\lambda+2\tau_a)}$	τ	-
F	$\frac{Ke^{-\theta s}}{s(\tau s+1)}$	$\frac{1}{K(\lambda+\theta)}$	-	$\frac{\theta}{3}$	$\frac{[\lambda\theta/3-\theta^2/6]}{(\theta+\lambda)}$	-	τ	-
G	$\frac{K(-\tau_a s+1)e^{-\theta s}}{s(\tau s+1)}$	$\frac{1}{K(\lambda+\theta+2\tau_a)}$	-	$\frac{\theta}{3}$	$\frac{\left[\lambda\tau_a+\frac{(\lambda+\tau_a)\theta}{3}-\theta^2/6-2\tau_a\theta/3\right]}{(\theta+\lambda+2\tau_a)}$	$\frac{\tau_a\theta\lambda/3+\tau_a\theta^2/6}{(\theta+\lambda+2\tau_a)}$	τ	-

Note: - In cases A, B, C, and E, desired closed-loop response $\frac{C}{R}=\frac{e^{-\theta s}}{\lambda s+1}$; in cases D, E, and G, $\frac{C}{R}=\frac{(-\tau_a s+1)e^{-\theta s}}{(\tau_a s+1)(\lambda s+1)}$

- In cases A, B, C, D, and E, the 2/2 Pade approximation is used; in cases F and G, the 2/1 Pade approximation is used.

proposed PID•filter structure. The truncation of the higher order terms has no impact on the performance of the resulting controller. The tuning formula derived is given in Table 1.

4. SOPDT and FOPDT Processes with Inverse Response

Consider an SOPDT model with a positive zero,

$$G_p=\frac{(-\tau_a s+1)Ke^{-\theta s}}{(\tau^2 s^2+2\tau\zeta s+1)} \tag{13}$$

The noninvertible portion becomes $P_A=(-\tau_a s+1)e^{-\theta s}/(\tau_a s+1)$ by applying the all-pass form for the non-minimum part $(-\tau_a s+1)$. Accordingly, the invertible portion is $P_M=K(\tau_a s+1)/(\tau^2 s^2+2\tau\zeta s+1)$. For the desired closed-loop response, $(-\tau_a s+1)e^{-\theta s}/(\lambda s+1)(\tau_a s+1)$, the ideal feedback controller is given as $G_c=(\tau^2 s^2+2\tau\zeta s+1)/(K[(\tau_a s+1)(\lambda s+1)-(\tau_a s+1)e^{-\theta s}])$. The PID•filter structure can be obtained in the same manner as follows in the SOPDT model with a lead term. The resulting tuning rule is given in Table 1.

The same procedure can be used to obtain the tuning rule for an FOPDT process with inverse response as,

$$G_p=\frac{K(-\tau_a s+1)e^{-\theta s}}{\tau s+1} \tag{14}$$

The resulting tuning rule is also given in Table 1.

5. First-order Delayed Integrating Processes

Consider a first-order delayed integrating process,

$$G_p=\frac{Ke^{-\theta s}}{s(\tau s+1)} \tag{15}$$

G_p is factored into $P_M=K/s(\tau s+1)$ and $P_A=e^{-\theta s}$. For the desired closed-loop response, $e^{-\theta s}/(\lambda s+1)$, the ideal controller can be converted to the PID•filter controller by approximating the dead time with a 2/1 Pade approximation and the resulting tuning rule is given in Table 1.

For the following first-order delayed integrating process with inverse response,

$$G_p=\frac{K(-\tau_a s+1)e^{-\theta s}}{s(\tau s+1)} \tag{16}$$

where G_p is decomposed as $P_M=K(\tau_a s+1)/s(\tau s+1)$, and $P_A=(-\tau_a s+1)e^{-\theta s}/(\tau_a s+1)$. The desired closed-loop response is set as $(-\tau_a s+1)e^{-\theta s}/(\lambda s+1)(\tau_a s+1)$. The PID•filter controller can be derived from the ideal feedback controller by using a 2/1 Pade approximation. The resulting tuning rule is given in Table 1. Note that the proposed IMC-PID approach essentially leads to a PD structure (with no integral action) for integrating processes.

ROBUST STABILITY

It is of significance in the proposed tuning rule to establish the relationship between the multiplicative uncertainty bound and the adjustable parameter λ. Robustness of the proposed controller can be analyzed theoretically by utilizing the robust stability theorem [3]. In the proposed tuning rule, λ can be adjusted for the uncertainties in the process and for the load disturbance.

Robust Stability Theorem (Morari and Zafirou, [3]): Assume that all plants G_p in the family Π

$$\Pi=\left\{G_p:\left|\frac{G_p(i\omega)-\tilde{G}_p(i\omega)}{\tilde{G}_p(i\omega)}\right|<\bar{l}_m(\omega)\right\} \tag{17}$$

have the same number of RHP poles and that a particular controller G_c stabilizes the nominal plant $\tilde{G}_p$. Then, the system is robustly stable with the controller G_c if and only if the complementary sensitivity function η for the nominal plant $\tilde{G}_p$ satisfies the following bound:

$$\|\tilde{\eta}\bar{l}_m\|_\infty = \sup_\omega |\tilde{\eta}\bar{l}_m(\omega)| < 1 \tag{18}$$

Since $\tilde{\eta} = \tilde{G}_p q = \tilde{G}_p p_m^{-1} f$ for the IMC controller, the resulting Eq. (18) becomes:

$$|\tilde{G}_p p_m^{-1} f \bar{l}_m(\omega)| < 1 \quad \forall \omega \tag{19}$$

Thus, the above theorem can be interpreted as $|\bar{l}_m| < 1/|\tilde{\eta}| = 1/|\tilde{G}_p p_m^{-1} f|$, which guarantees robust stability when the multiplicative model error is bounded by $|\Delta_m(s)| \le \bar{l}_m$.

$$\|\tilde{\eta}(s)\Delta_m(s)\|_\infty < 1 \tag{20}$$

where $\Delta_m(s)$ defines the process multiplicative uncertainty bound. *i.e.*, $\Delta_m(s)=(G_p-\tilde{G}_p)/\tilde{G}_p$. This uncertainty bound can be utilized to represent the model reduction error, process input actuator uncertainty, and process output sensor uncertainty, etc., which often occur in actual process plants.

For an FOPDT process, the complementary sensitivity function $\eta(s)$ is given as

$$\tilde{\eta}(s) = \frac{e^{-\theta s}}{(\lambda s+1)} \tag{21}$$

Substituting Eq. (21) into Eq. (20) yields the robust stability constraint required for tuning the adjustable parameters λ.

$$\left\|\frac{1}{(\lambda s+1)}\right\|_\infty < \frac{1}{\|\Delta_m(s)\|_\infty} \tag{22}$$

Let us consider the first-order stable process with dead time having the uncertainty in all three parameters as follows

$$G_p = \frac{(K+\Delta K)e^{-(\theta+\Delta\theta)s}}{(\tau s+1)(\Delta\tau s+1)} \tag{23}$$

where Δ is the multiplicative uncertainty on each process parameter.

Since a low-order model approximates a high-order process, the small time constant $\Delta\tau$ is assumed to be neglected/missing in developing the nominal model as considered in Eq. (23). Then the process multiplicative uncertainty bound becomes

$$\Delta_m(s) = \frac{\left(1+\frac{\Delta K}{K}\right)e^{-\Delta\theta s}}{(\Delta\tau s+1)} - 1 \tag{24}$$

Substituting Eq. (24) into Eq. (22) and then s=iω in the resulting equation, we obtain the robust stability constraint as follows:

$$\frac{1}{\sqrt{(\lambda^2\omega^2+1)}} < \frac{1}{\left|\frac{\left(1+\frac{\Delta K}{K}\right)e^{-\Delta\theta i\omega}}{(\Delta\tau i\omega+1)} - 1\right|}, \quad \forall \omega > 0 \tag{25}$$

The robust stability constraint in Eq. (25) is very useful to adjust λ where there is uncertainty in the process parameters. It can also be useful for the determination of the maximum allowable values of uncertainty in $\pm\Delta K$, $\pm\Delta\theta$ and $\pm\Delta\tau$ or various combinations of them for which robust stability can be guaranteed. For an example, a plot of $|\tilde{\eta}(\omega)\bar{l}_m(\omega)|$ vs. ω can be constructed for a small value of any parametric uncertainty and/or combination of different uncertainties. A similar stability and robustness analysis can be carried out for the other process models as well.

SIMULATION STUDIES

To evaluate the robustness of a control system, the maximum sensitivity, Ms, which is defined by Ms=max$|1/[1+G_pG_c(i\omega)]|$, is used. Since Ms is the inverse of the shortest distance from the Nyquist curve of the loop transfer function to the critical point (−1, 0), a small value indicates that the stability margin of a control system is large. Typical values of Ms are in the range of 1.2-2.0 as recommended by Åström and Hägglund [12]. For fair comparison under the same robustness level, throughout all our simulation examples all the controllers compared were designed to have the same Ms value of Ms=1.60. To evaluate the closed-loop performance, two performance indices were considered to a step set-point change and a step load disturbance: the integral of the time-weighted absolute error (ITAE), defined by ITAE=$\int_0^\infty t|e(t)|dt$, and the overshoot as a measure of how much the response exceeds the ultimate value following a step change in set-point and/or disturbance. Robust performance was also evaluated in all examples by perturbing +20% uncertainties in the gain and the dead time.

Example 1: Consider the following FOPDT process studied by Lee et al. [6].

$$G_p = \frac{1e^{-3s}}{10s+1} \tag{26}$$

Conventional PID controllers based on the methods of Lee et al. [6], Rivera et al. [2] are used for comparison. For the proposed method, Lee et al.'s [6] method and Rivera et al.'s [2] method, λ values of 2.031, 1.774, and 2.592, respectively, are selected so that every controller satisfies Ms=1.60. The closed-loop responses of the three controllers for the nominal case are shown in Fig. 2. The values of controller parameters and resulting performance indices are listed in Table 2. The simulation results indicate that the proposed PID•filter controller provides a fast and smooth set-point response without any loss of disturbance performance.

The recently published tuning method of Shamsuzzoha and Lee

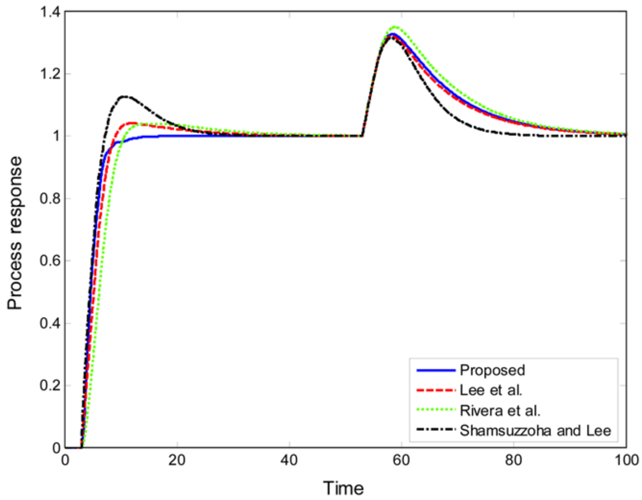

Fig. 2. Simulation result of proposed tuning method for Example 1.

Table 2. Controller parameters and resulting performance indices for the FOPDT process (Example 1)

Method	K_c	τ_I	τ_D	a	b	c	Set-point				Disturbance			
							Nominal case		20% mismatch		Nominal		20% mismatch	
							ITAE	Overshoot	ITAE	Overshoot	ITAE	Overshoot	ITAE	Overshoot
Proposed λ=2.031	0.298	1.5	0.50	0.605	0.302	10	14.09	0	27.33	0.25	80.06	0.32	75.86	0.41
Lee et al. λ=1.775	2.292	10.94	0.856	-	-	-	27.30	0.04	36.86	0.27	75.02	0.32	71.17	0.40
Rivera et al. λ=2.592	2.057	11.5	1.304	0.695	-	-	39.31	0.03	47.78	0.24	91.58	0.35	86.38	0.44
Shamsuzzoha and Lee λ=3.764	0.440	1.5	0.5	0.601	-	7.1	28.91	0.127	40.42	0.39	38.94	0.314	37.01	0.40

Table 3. Controller parameters and resulting performance indices for the underdamped SOPDT process (Example 2)

Method	K_c	τ_I	τ_D	a	b	c	d	Set-point				Disturbance			
								Nominal case		20% mismatch		Nominal		20% mismatch	
								ITAE	Overshoot	ITAE	Overshoot	ITAE	Overshoot	ITAE	Overshoot
Proposed λ=2.707	0.298	2.0	0.666	0.807	0.538	10	100	24.8	0.0	49.6	0.24	271.9	0.34	275.5	0.39
Lee et al. λ=2.1105	1.268	10.43	9.893	-	-	-	-	392.7	0.28	411.3	0.35	322.8	0.43	326.60	0.50
Chien and Fruehauf λ=3.91	1.264	10.0	10.0	-	-	-	-	424.7	0.30	441.6	0.37	333.2	0.43	337.50	0.50

[14] has been developed for the improved disturbance rejection. Note that any PID controller design method for improved disturbance rejection causes overshoots in the setpoint response. For such a case, the setpoint filter is suggested to enhance the servo response. For illustration purpose, the proposed design method, which is based on the improved setpoint response, has been also compared with the Shamsuzzoha and Lee [14]. The result is also included in Table 2 for the same robustness level as the other method. Table 2 and Fig. 2 show that the proposed design method gives significant advantage for the servo performance for the single degree of freedom controller, whereas the setpoint performance of Shamsuzzoha and Lee [14] can be improved by including the setpoint filter.

The robust performance of the proposed controller is evident in the results of the model mismatch case in Table 2. The proposed controller still gives the smallest ITAE in the set-point response.

Example 2: Consider the following underdamped SOPDT process model

$$G_p = \frac{1e^{-4s}}{(100s^2+10s+1)} \quad (27)$$

where ζ=0.5. The proposed PID•filter controller is compared with the conventional PID controllers based on the methods of Lee et al. [6] and Chien and Fruehauf [4]. All controllers are tuned to have Ms=1.60 by adjusting λ. The resulting parameter settings and performance indices are listed in Table 3. Fig. 3 compares the closed-loop responses achieved by the three controllers. The results show that the proposed controller exhibits significant advantage over the other two controllers not only in terms of servo performance but also in load performance. The proposed controller also shows superior robust performance in the model-mismatch case.

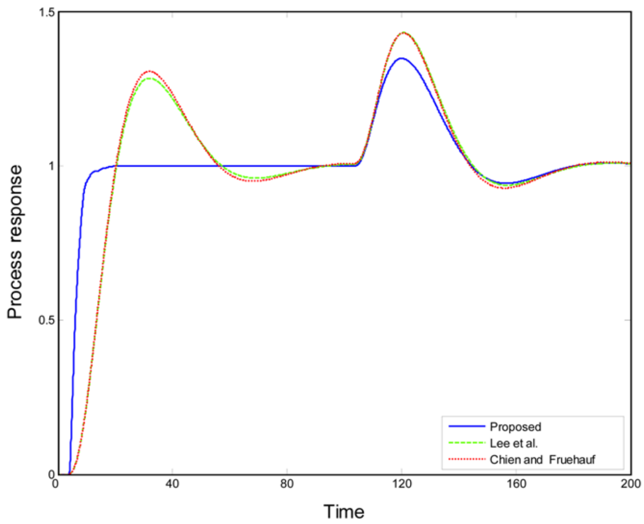

Fig. 3. Simulation result of proposed tuning method for Example 2.

Example 3: Consider the overdamped SOPDT process studied by Seborg et al. [13].

$$G_p = \frac{2e^{-1s}}{(10s+1)(5s+1)} \quad (28)$$

Table 4. Controller parameters and resulting performance indices for the overdamped SOPDT process (Example 3)

Method	K_c	τ_I	τ_D	a	b	c	d	Set-point				Disturbance			
								Nominal case		20% mismatch		Nominal		20% mismatch	
								ITAE	Overshoot	ITAE	Overshoot	ITAE	Overshoot	ITAE	Overshoot
Proposed λ=0.6766	0.149	0.5	0.166	0.202	0.034	15	50	1.55	0.0	3.08	0.25	56.88	0.17	55.95	0.17
Lee et al. λ=0.515	3.723	15.11	3.418	-	-	-	-	73.79	0.12	71.36	0.12	69.57	0.20	68.2	0.20
Smith et al. λ=0.9773	3.793	15.0	3.333	-	-	-	-	71.56	0.12	69.06	0.13	67.08	0.20	65.77	0.20
Chien and Fruehauf λ=0.9773	3.793	15.0	3.333	-	-	-	-	71.56	0.12	69.06	0.13	67.08	0.20	65.77	0.20
Shamsuzzoha and Lee λ=1.521	6.908	6.6059	1.941	0.047	-	0.50	-	25.69	0.33	23.79	0.32	7.29	0.12	7.14	0.13

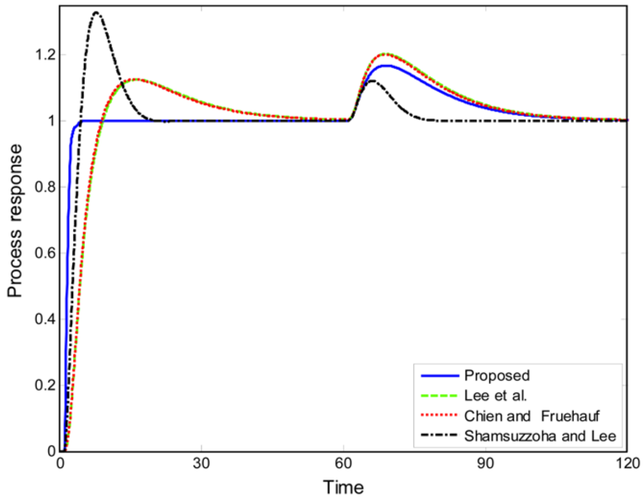

Fig. 4. Simulation result of proposed tuning method for Example 3.

The proposed controller was compared with the PID controllers based on the methods of Lee et al. [6], Chien and Fruehauf [4] and Shamsuzzoha and Lee [8]. All controllers are designed to meet Ms=1.60. The performance indices in Table 4 show the superiority of the proposed PID•filter controller in both model mismatch and nominal cases. Fig. 4 shows the closed-loop responses of the three controllers in the nominal case. It is important to mention that the controller design method of Shamsuzzoha and Lee [8] is based on the enhanced disturbance rejection and it causes a significant overshoot in the setpoint response. The proposed design method shows improved servo performance while maintaining the disturbance rejection. As seen from the figure, the proposed controller has no overshoot and negligible settling time compared with the other two controllers.

Example 4: Simulation study was carried out for the following SOPDT model with a strong lead term

$$G_p = \frac{(15s+1)e^{-6s}}{(10s+1)(10s+1)} \tag{29}$$

The proposed PID•filter controller is compared with the conventional PID controllers based on the methods of Lee et al. [6] and Chien and Fruehauf [4]. All controllers are tuned to have the same robustness as Ms=1.60. The controller parameter settings and the resulting values of the performance indices are listed in Table 5. The closed-loop responses of the three controllers are compared in Fig. 5. The results show that the proposed controller gives a superior performance both for set-point and load responses over the other two controllers. The proposed controller is also robust in the face of model uncertainty.

Example 5: Consider the following SOPDT process with inverse

Table 5. Controller parameters and resulting performance indices for the second order delayed overshoot process (Example 4)

Method	K_c	τ_I	τ_D	a	b	c	d	Set-point				Disturbance			
								Nominal case		20% mismatch		Nominal		20% mismatch	
								ITAE	Overshoot	ITAE	Overshoot	ITAE	Overshoot	ITAE	Overshoot
Proposed λ=3.666	0.310	3.0	1.0	15.827	12.412	20	100	53.37	0.0	121.3	0.34	188.7	0.62	185.6	0.95
Lee et al. λ=6.544	0.513	6.434	4.874	-	-	-	-	184.6	0.07	184.4	0.15	320.5	0.62	340.5	0.95
Chien and Fruehauf λ=3.999	0.50	5.0	5.0	-	-	-	-	245.3	0.16	286.7	0.27	323.3	0.62	397.7	0.95

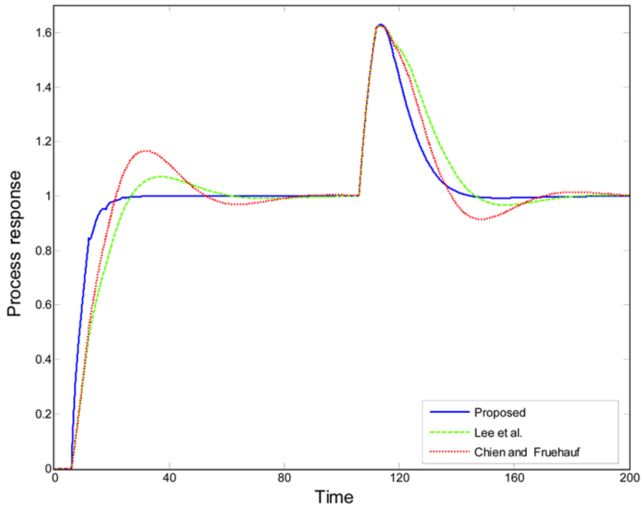

Fig. 5. Simulation result of proposed tuning method for Example 4.

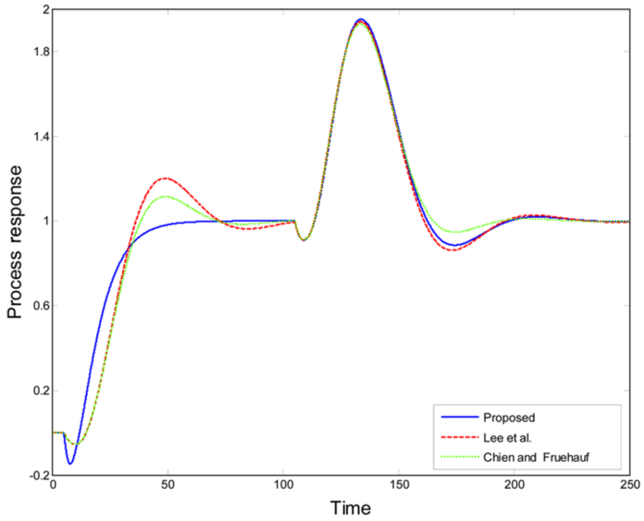

Fig. 6. Simulation result of proposed tuning method for Example 5.

response

$$G_p=\frac{(-5s+1)e^{-5s}}{(100s^2+10s+1)} \tag{30}$$

The proposed PID•filter controller is compared with the conventional PID controllers based on the methods of Lee et al. [6] and Chien and Fruehauf [4], while all controllers are tuned to meet Ms=1.60. The controller parameter settings and the resulting values of the performance indices are listed in Table 6. The closed-loop responses of the three controllers are shown in Fig. 6. The superior performance achieved by the proposed controller is readily apparent from the figure.

Example 6: Set-point response was studied for the following first-order delayed integrating process

$$G_p=\frac{1e^{-5s}}{s(10s+1)} \tag{31}$$

As shown in Fig. 7 and Table 7, the proposed controller shows superior performance over the other two controllers.

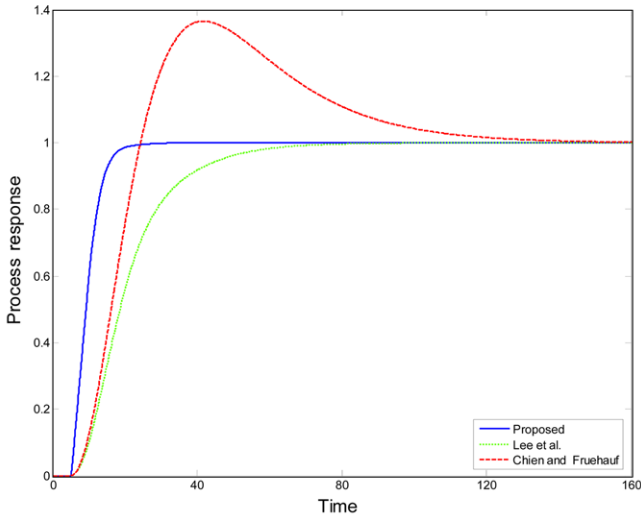

Fig. 7. Simulation result of proposed tuning method for Example 6.

DISCUSSION

Consider a lag time dominant FOPDT process where $\theta \ll \tau$ and λ-τ. Using a 2/2 Pade approximation for the dead time, the ideal feedback controller to give the desired closed-loop response, $e^{-\theta s}/(\lambda s+1)$, can be converted to the PID•filter controller as:

$$G_c=\frac{(\theta^2s^2/12+\theta s/2+1)}{K(\lambda+\theta)s}\bullet\frac{(\tau s+1)}{\left[\frac{\theta^2\lambda}{12(\lambda+\theta)}s^2+\frac{\theta\lambda}{2(\lambda+\theta)}s+1\right]} \tag{32}$$

Table 6. Controller parameters and resulting performance indices for the second order delayed inverse process (Example 5)

Method	K_c	τ_I	τ_D	a	b	c	d	Set-point				Disturbance			
								Nominal case		20% mismatch		Nominal		20% mismatch	
								ITAE	Overshoot	ITAE	Overshoot	ITAE	Overshoot	ITAE	Overshoot
Proposed λ=8.44	0.106	2.50	0.833	2.611	4.945	10	100	309.5	0.0	384.0	0.15	1181	0.95	1876.0	1.26
Lee et al. λ=7.381	0.448	10.026	9.628	-	-	-	-	666.7	0.20	1153.0	0.46	1208	0.94	2000.0	1.25
Chien and Fruehauf λ=14.512	0.449	11.019	10.094	-	-	-	-	509.2	0.11	781.5	0.34	1031	0.93	1487.0	1.24

Table 7. Controller parameters and resulting performance indices for the first order delayed integrating process (Example 6)

Method	K_c	τ_I	τ_D	a	c	Set-point			
						Nominal case		20% mismatch	
						ITAE	Overshoot	ITAE	Overshoot
Proposed λ=3.156	0.122	-	1.666	0.134	10	54.63	0.0	83.39	0.18
Lee et al. λ=5.5455	0.094	-	11.185	-	-	314.40	0.0	258.6	0.0
Chien and Fruehauf λ=15.299	0.110	45.598	7.806	-	-	1054.0	0.36	960.7	0.42

The loop transfer function then becomes:

$$L(s)=G_cG_p=\frac{1}{(\lambda+\theta)s}\bullet\frac{1}{\left[\frac{\theta^2\lambda}{12(\lambda+\theta)}s^2+\frac{\theta\lambda}{2(\lambda+\theta)}s+1\right]} \quad (33)$$

under the assumption that the process dead time $e^{-\theta s}$ is effectively compensated for by the lead term $(1+\theta s/2+\theta^2s^2/12)$ in the lead-lag filter. It is clear that the PID controller compensates the dominant pole in the process transfer function effectively. Consequently, the resulting control system is equivalent to controlling the fast process $1/[(\theta^2\lambda)/12(\lambda+\theta)s^2+(\theta\lambda)/(2(\lambda+\theta))s+1]$ with the integral controller. Note that as λ increases, the PID•filter controller approaches the conventional PID controller.

For a lag time dominant SOPDT process, the ideal feedback controller is converted to the PID•filter controller as:

$$G_c=\frac{(\theta^2s^2/12+\theta s/2+1)}{K(\lambda+\theta)s}\bullet\frac{(\tau^2s^2+2\tau\xi s+1)}{\left[\frac{\theta^2\lambda}{12(\lambda+\theta)}s^2+\frac{\theta\lambda}{2(\lambda+\theta)}s+1\right]} \quad (34)$$

The loop transfer function is exactly the same as Eq. (33) by the FOPDT model.

Since the resulting filter has the damping factor $\xi_f=\sqrt{3\lambda/4(\lambda+\theta)}$, it is always underdamped. The maximum filter value can have is 0.866 when $\theta\ll\lambda$. As the λ value decreases, the underdamping of the filter increases, which can cause a severe robustness problem.

Fig. 8 compares the closed-loop responses by various controllers for the previously studied process $G_p=2e^{-1s}/(10s+1)(5s+1)$. The approximated loop transfer function by Eq. (33) is (for λ=0.6766):

$$L(s)=\frac{1}{1.6766s}\frac{1}{(0.336s^2+0.2018s+1)} \quad (35)$$

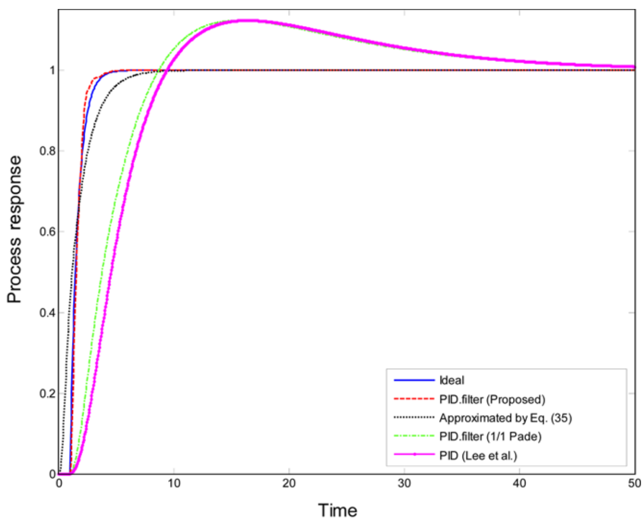

Fig. 8. Closed-loop responses by various controllers.

In the figure, the PID•filter (1/1Pade) is derived by using the 1/1 Pade approximation and consists of the PID controller cascaded with a first-order lead-lag filter. As seen in the figure, the response by the proposed PID•filter controller follows the response by the ideal controller very closely, while that by the PID•filter (1/1Pade) controller is quite different. This indicates that the proposed PID•filter controller based on the 2/2 Pade expansion approximates the ideal controller almost perfectly, while the 1/1 Pade expansion does not. Furthermore, the response based on the approximated loop transfer function by Eq. (35) shows quite a similar trend with that by the PID•filter controller, which indicates that the lead term of the filter plays an important role in improving closed-loop performance by compensating for the dead time effect.

The proposed PID•filter controller has a clear advantage as the lag time dominates. Fig. 9 compares the ITAE values of the set-point responses for various dead time to lag time ratios for the FOPDT

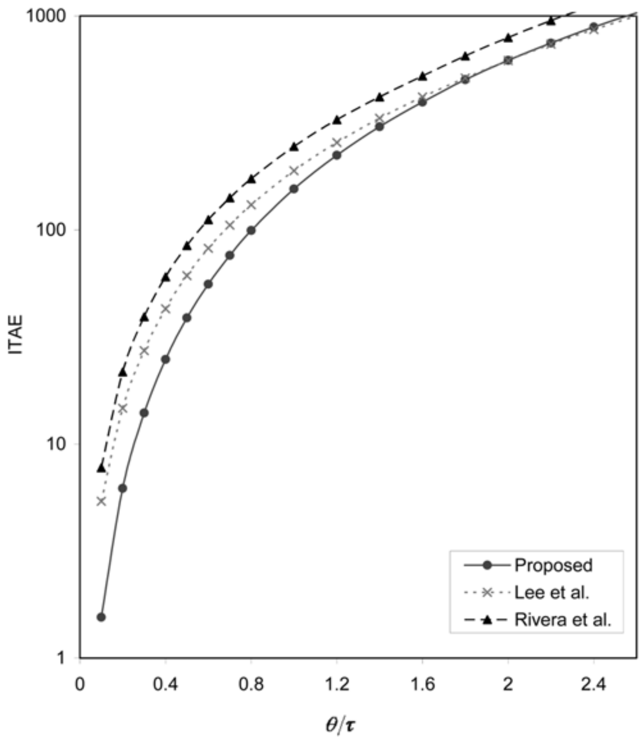

Fig. 9. Comparison of the ITAE generated by various tuning rules for Ms=1.60.

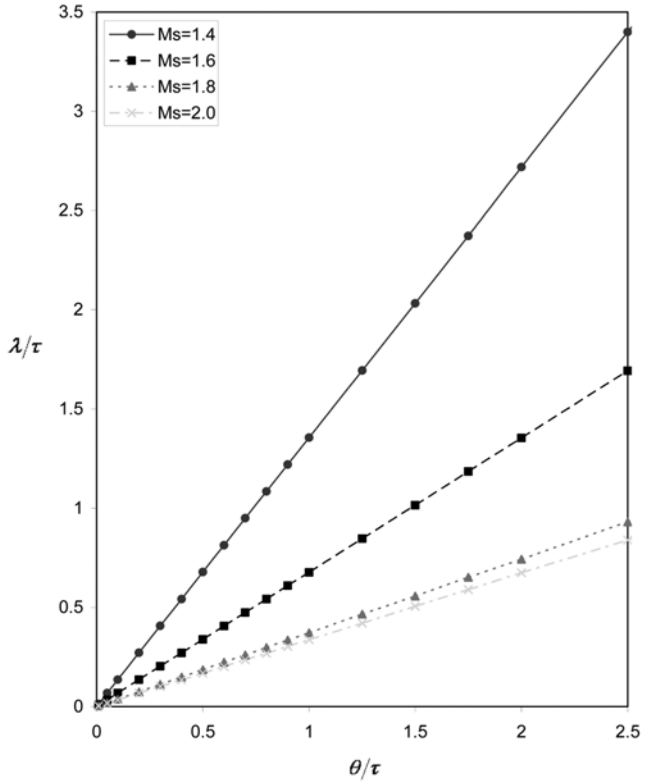

Fig. 10. λ guide lines for FOPDT and SOPDT for different Ms values.

model used in Example 1 (by changing θ while fixing τ). λ is chosen so that Ms=1.6 for every case. As seen from the figure, the proposed PID•filter controller gives the smallest ITAE value among all other controllers over the lag time dominant range. As θ/τ increases (i.e., the dead time dominates), the benefit gained by the proposed PID•filter controller is diminished. In Fig. 9, the conventional PID controller based on the methods of Lee et al. [6] gives the smallest ITAE value over the other controllers when θ/τ goes beyond 2.

λ GUIDELINE

In the proposed tuning rule, the closed-loop time constant λ controls the tradeoff between robustness and performance of the control system. As λ decreases, the closed-loop response becomes faster and can become unstable. On the other hand, as λ increases, the closed-loop response becomes sluggish and stable. A good tradeoff is obtained by choosing λ to give an Ms value in the range of 1.2-2.0. The λ guideline plot for several robustness levels is shown in Fig. 10. As seen from the figure, the resulting plot is almost a straight line and the desired λ value can also be obtained from a linear relation by $\lambda/\tau=\alpha(\theta/\tau)$, where α=1.360, 0.677,0.372, and 0.337 for Ms=1.4, 1.6, 1.8, and 2.0, respectively. For small θ/τ (typically less than 0.2), a detuning process might be considered to account for the constraint on the manipulated variable. Since the λ value for a particular Ms is independent on the damping factor in the process model, the plot in Fig. 10 can also be used for λ setting of the SOPDT model.

CONCLUSIONS

An analytical design method for a PID controller in series with a second-order lead-lag filter is proposed for various types of time-delay process. By using the appropriate Pade expansion to the process dead time, the ideal feedback controller can be converted into a PID•filter structure with little loss of accuracy. The resulting PID•filter controller efficiently compensates for the dominant process poles and zeros and significantly improves the closed-loop performance. Simulation results demonstrate the superior performance of the proposed PID•filter controller over the conventional PID controllers. A guideline of closed-loop time constant is also proposed at a several Ms values for a wide range of θ/τ ratio for FOPDT and SOPDT process.

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